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## Experimental electron density of urea-phosphoric $\operatorname{acid}(1 / 1)$ at 100 K

The deformation electron density of the urea-phosphoric acid adduct has been studied from 100 K X-ray and neutron diffraction experiments. Data were interpreted according to the Hirshfeld model. The long hydrogen bonds show characteristics of electrostatic interaction. Deformation density maps on the short hydrogen bond shows hydrogen more strongly bonded to urea than to phosphoric acid, and peak maxima at almost midway between the two $\mathrm{O}-\mathrm{H}$ bonds.

## 1. Introduction

Urea was the subject of two very accurate experimental electron density studies. One of these (Swaminathan, Craven \& McMullan, 1984) was performed at 123 K and illustrated the efficiency of deformation models to describe this noncentrosymmetric system. The other study (Zavodnik et al., 1999) was performed with data collected at 148 K and discussed the importance of correcting data for thermal diffuse scattering. The electron density of phosphoric acid has also been the subject of a very accurate experimental study (Souhassou et al., 1995), which shows the influence of hydrogen bonding on the polarization of $\mathrm{P}-\mathrm{O}$ bonds. These multipole studies agree well with $a b$ initio calculations for crystalline urea (Dovesi et al., 1990) and phosphoric acid (Moss et al., 1995). The crystal structures of urea (Swaminathan, Craven, Spackman \& Stewart, 1984) and phosphoric acid (Blessing, 1988) were also determined by neutron diffraction.

Reaction between urea and phosphoric acid gives a solid adduct. Previous works based on room-temperature X-ray (Motz \& Albrand, 1972) and neutron (Konstansek \& Busing, 1972) data show that the crystalline adduct is stabilized by many hydrogen bonds, including a short intramolecular $\mathrm{O}-$ $\mathrm{H} \cdots \mathrm{O}$ bond $[d(\mathrm{O} \cdots \mathrm{O})=2.421 \AA$ from neutron data]. Previous X-ray and neutron (X-N) studies of the adduct (Savage et al., 1987) suggest that the short hydrogen bond should be interpreted as a three-centre, four-electron bond. Therefore, an accurate study on the electron density of ureaphosphoric acid $\left[\mathrm{H}_{3} \mathrm{PO}_{4} \cdot \mathrm{CO}\left(\mathrm{NH}_{2}\right)_{2}\right]$ can contribute to the understanding of the nature of short hydrogen bonds. We present here an experimental electron density study of $\mathrm{H}_{3} \mathrm{PO}_{4} \cdot \mathrm{CO}\left(\mathrm{NH}_{2}\right)_{2}$ by neutron and high-resolution X-ray $\left(\sin \theta / \lambda \leq 1.19 \AA^{-1}\right.$ ) diffraction methods. Data were collected at 100 K , since thermal vibrational effects are less correlated to electron density parameters at low temperatures (Hirshfeld, 1976).

Table 1
Experimental details.

|  | Neutron | X-ray |
| :---: | :---: | :---: |
| Crystal data |  |  |
| Chemical formula | $\mathrm{CH}_{7} \mathrm{~N}_{2} \mathrm{O}_{5} \mathrm{P}$ | $\mathrm{CH}_{7} \mathrm{~N}_{2} \mathrm{O}_{5} \mathrm{P}$ |
| Chemical formula weight | 159 | 159 |
| Cell setting, space group | Orthorhombic, Pbca | Orthorhombic, Pbca |
| $a, b, c$ ( ${ }_{\text {® }}$ ) | $\begin{aligned} & 17.43(2), 7.43(2), \\ & 8.97(2) \end{aligned}$ | $\begin{aligned} & 17.4527 \text { (7), } 7.4465 \text { (5), } \\ & 8.9589 \text { (4) } \end{aligned}$ |
| $V\left(\AA^{3}\right)$ | 1161 (4) | 1164.31 (9) |
| $Z$ | 8 | 8 |
| $D_{x}\left(\mathrm{Mg} \mathrm{m}^{-3}\right)$ | 1.820 | 1.815 |
| Radiation type | Neutron | Mo $K \alpha$ |
| Wavelength ( $\AA$ ) | 1.207 | 0.71073 |
| No. of reflections for cell parameters | 9 | 98 |
| $\theta$ range ( ${ }^{\circ}$ ) | 18.9-30.9 | 12-20 |
| $\mu\left(\mathrm{mm}^{-1}\right)$ | 2.225 (obs.) | 0.431 (calc.) |
| Temperature (K) | 100 | 100 |
| Crystal form, colour | Prismatic, colourless | Almost spherical, colourless |
| Crystal size (mm) | $3.8 \times 2.2 \times 1.0$ | - |
| Crystal radius (mm) | - | 0.19 |
| Data collection |  |  |
| Diffractometer | Huber four-circle | Siemens P4 |
| Data collection method | $\theta-2 \theta$ scans | $\theta-2 \theta$ scans |
| Absorption correction | Analytical | Analytical |
| $T_{\text {min }}$ | 0.55 | 0.87 |
| $T_{\text {max }}$ | 0.66 | 0.89 |
| No. of measured, independent and observed parameters | 1762, 1599, 1579 | 8472, 7433, 7257 |
| Criterion for observed reflections | $I>-3 \sigma(I)$ | $I>0.0 \sigma(I)$ |
| $R_{\text {int }}$ | 0.017 | 0.0134 |
| $\theta_{\text {max }}\left({ }^{\circ}\right)$ | 52 | 60 |
| Range of $h, k, l$ | $-22 \rightarrow h \rightarrow 0$ | $-1 \rightarrow h \rightarrow 41$ |
|  | $-9 \rightarrow k \rightarrow 0$ | $-1 \rightarrow k \rightarrow 17$ |
|  | $0 \rightarrow l \rightarrow 11$ | $-1 \rightarrow l \rightarrow 21$ |
| Intensity decay (\%) | 0 | 1.1 |
| Refinement |  |  |
| Refinement on | $F^{2}$ | $F^{2}$ |
| $R\left[F^{2}>2 \sigma\left(F^{2}\right)\right], w R\left(F^{2}\right), S$ | 0.053, 0.053, 1.35 | 0.0577, 0.053, 0.69 |
| No. of reflections and parameters used in refinement | 1579, 151 | 7257, 215 |
| H -atom treatment | Refined anisotropically | Hydrogen fixed from neutron |
| Weighting scheme | $w=1 /\left[\sigma^{2}\left(F_{o}^{2}\right)\right]$ | $\begin{aligned} w & =1 /\left[\sigma^{2}\left(F_{o}^{2}\right)+0.0068 F_{o}^{2}\right. \\ & +34.9 \lambda / \sin \theta \\ & \left.-0.00055 F_{o}^{2} \lambda / \sin (\theta)\right] \end{aligned}$ |
| $(\Delta / \sigma)_{\text {max }}$ | 0.001 | 0.001 |
| $\Delta \rho_{\text {max }}, \Delta \rho_{\text {min }}$ (nuclei A ) | $0.38,-0.19$ | $0.61,-0.62$ |
| Extinction method | See text | See text |

Computer programs used: XSCANS (Siemens, 1991), P4red, DUPALS (Lundgren, 1982), ORTEP3 (Burnet \& Johnson, 1996), FOPLOT (Lundgren, 1982), ARACOR (Lundgren, 1982), Ms-DOS 6.0 editor.

### 2.2. Data collection

Neutron diffraction data were collected at the Swedish Research Reactor, R2, in Studsvik, with a flux at the crystal location of approximately $1.15 \times 10^{4}$ neutron $\mathrm{mm}^{-2} \mathrm{~s}^{-1}$. Experimental details are given in Table 1. ${ }^{\mathbf{1}}$ Lorentz and absorption corrections were applied using the program package of the Institute of Chemistry, Uppsala University (Lundgren, 1982).

X-ray data were collected on a Siemens P4 diffractometer equipped with a lowtemperature device. Data collection features are given in Table 1. Data were corrected by Lorentz, polarization and absorption effects. This latter correction was performed analytically by Gaussian numerical quadrature over a $6 \times 6 \times 6$ grid.

## 3. Refinements

### 3.1. Neutron data

Neutron scattering lengths given in International Tables for Crystallography (Sears, 1995) were used. Starting from roomtemperature neutron diffraction results (Konstansek \& Busing, 1972), the positional and isotropic thermal displacement parameters of non-H atoms were refined using the 1579 observed reflections. Then, H atoms were located in the Fourier-difference synthesis and anisotropic displacement parameters for all atoms were introduced. This refinement resulted in non-positive displacement parameters for P and C atoms. Isotropic and anisotropic extinction models of Becker \& Coppens (1974, 1975) were applied to solve this problem. The best results were obtained with anisotropic type I extinction and Lorentzian distribution of mosaic blocks. Therefore, in the final neutron refinement 144 atomic and 6 anisotropic extinction parameters, as well as the scale factor, were refined, resulting in 151 refined

## 2. Experimental

### 2.1. Crystal growth

Solid $\mathrm{H}_{3} \mathrm{PO}_{4} \cdot \mathrm{CO}\left(\mathrm{NH}_{2}\right)_{2}$ is obtained from an equimolar aqueous solution of urea and phosphoric acid. Recrystallization in methanol gives single crystals suitable for X-ray diffraction experiments. One of these was chosen and measured on a Siemens P4 diffractometer equipped with a nitrogen cryosystem. Another was selected as a seed to grow the single crystal used in the neutron data collection procedure. The sample was obtained by slow evaporation of a saturated methanolic solution.
parameters. The drawing of the molecule is shown in Fig. 1 and the main results of the refinement are given in Table 1.

### 3.2. X-ray data

3.2.1. Conventional refinements and extinction correction. The refinements started from positional and displacement parameters obtained from neutron diffraction data. X-ray scattering factors given by Maslen et al. (1995) were used. First

[^0]of all isotropic extinction models of Becker and Coppens were tested. During the extinction tests, positional and anisotropic thermal displacement parameters of non-H atoms were refined, as well as the scale factor and the extinction parameter. All extinction correction models gave negative extinction parameters and did not improve the statistical errors. Therefore, extinction corrections for X-ray diffraction data were not considered.
3.2.2. Multipole refinements. Positional and displacement parameters obtained from neutron diffraction data are used as a starting point for multipole refinements. These refinements consider the asphericity of electron distribution as an expansion of functions according to the method proposed by Hirshfeld (1971) and modified by Harel \& Hirshfeld (1975), and Hirshfeld (1977). The model expresses the non-sphericity of electron distribution by an expansion of up to 35 terms centred on each atom. In this way, the expansion functions have the following general form on each atomic centre
\[

$$
\begin{aligned}
\delta \rho_{a} & =\sum_{l} c_{a, l} \rho_{a, l}, \text { with } \\
\rho_{a, l}(\mathbf{r}) & =\rho_{n}\left(r_{a}, \theta_{k}\right)=N_{n} r^{n} e^{-\gamma r^{m}} \cos ^{n} \theta_{k},
\end{aligned}
$$
\]

where $r$ and $\theta$ are polar coordinates in the $k$ th of a chosen set of axes; $n$ and $k$ are integers $[n=0-4 ; k=1$ to $(n+1)(n+2) / 2]$; $m$ is 1 or 2 , indicating exponential or Gaussian dependence of the radial part of the expansion functions $\delta \rho_{a} ; N_{n}$ is a normalization factor and $l$ is a summation of $k$ terms up to $n=$ 4. The method refines parameters $c_{a, l}$.

Functions with $n \leq 4$ for non- H atoms and H 4 were used in the least-squares refinement. Functions with $n \leq 3$ were used for all other H atoms. The best results were obtained assuming Gaussian dependence of radial functions ( $m=2$ ). Local symmetries have been adopted in order to reduce the number of refined parameters. In this way, the multipole refinement started with the following local symmetry on each atom: mm 2 for $\mathrm{P}, \mathrm{O} 2, \mathrm{O} 4, \mathrm{O} 5, \mathrm{~N} 1, \mathrm{~N} 2$ and C ; $m$ for O 1 and O3; cylindrical for all hydrogens except H 4 , for which an inversion centre was added to cylindrical symmetry. O1 and O3 were assumed to be chemically equivalent, as well as N 1 and N 2 . H atoms bonded


Figure 1
ORTEP3 (Burnet \& Johnson, 1996) stereoscopic view of the molecule and its hydrogen-bonded neighbours. The ellipsoids correspond to $50 \%$ probability.

Table 2
Bond distances ( $\AA$ ) and angles $\left({ }^{\circ}\right)$.

| $\mathrm{P}-\mathrm{O} 1$ | $1.562(2)$ | $\mathrm{O} 1-\mathrm{H} 1$ | $1.000(3)$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{P}-\mathrm{O} 2$ | $1.512(2)$ | $\mathrm{O} 3-\mathrm{H} 3$ | $1.005(3)$ |
| $\mathrm{P}-\mathrm{O} 3$ | $1.554(2)$ | $\mathrm{N} 1-\mathrm{H} 11$ | $1.016(3)$ |
| $\mathrm{P}-\mathrm{O} 4$ | $1.517(2)$ | $\mathrm{N} 1-\mathrm{H} 12$ | $1.004(3)$ |
| $\mathrm{C}-\mathrm{N} 1$ | $1.323(1)$ | $\mathrm{N} 1-\mathrm{H} 12$ | $1.017(3)$ |
| $\mathrm{C}-\mathrm{N} 2$ | $1.325(1)$ | $\mathrm{N} 2-\mathrm{H} 22$ | $1.003(3)$ |
| $\mathrm{C}-\mathrm{O} 5$ | $1.295(2)$ |  |  |
|  |  |  | $113.5(2)$ |
| $\mathrm{O} 1-\mathrm{P}-\mathrm{O} 2$ | $111.4(1)$ | $\mathrm{P}-\mathrm{O} 1-\mathrm{H} 1$ | $117.0(2)$ |
| $\mathrm{O} 1-\mathrm{P}-\mathrm{O} 3$ | $108.3(1)$ | $\mathrm{P}-\mathrm{O} 3-\mathrm{H} 3$ | $125.1(2)$ |
| $\mathrm{O} 1-\mathrm{P}-\mathrm{O} 4$ | $105.5(1)$ | $\mathrm{P}-\mathrm{O} 4-\mathrm{H} 4$ | $118.6(2)$ |
| $\mathrm{O} 2-\mathrm{P}-\mathrm{O} 3$ | $106.6(1)$ | $\mathrm{C}-\mathrm{N} 1-\mathrm{H} 11$ | $120.2(2)$ |
| $\mathrm{O} 2-\mathrm{P}-\mathrm{O} 4$ | $113.6(1)$ | $\mathrm{C}-\mathrm{N} 1-\mathrm{H} 12$ | $120.8(2)$ |
| $\mathrm{O} 3-\mathrm{P}-\mathrm{O} 4$ | $111.3(1)$ | $\mathrm{H} 11-\mathrm{N} 1-\mathrm{H} 12$ | $120.4(2)$ |
| $\mathrm{O} 5-\mathrm{C}-\mathrm{N} 1$ | $118.3(1)$ | $\mathrm{C}-\mathrm{N} 2-\mathrm{H} 21$ | $120.5(2)$ |
| $\mathrm{O} 5-\mathrm{C}-\mathrm{N} 2$ | $121.0(1)$ | $\mathrm{C}-\mathrm{N} 2-\mathrm{H} 22$ | $119.0(2)$ |
| $\mathrm{N} 1-\mathrm{C}-\mathrm{N} 2$ | $120.8(1)$ | $\mathrm{H} 21-\mathrm{N} 2-\mathrm{H} 22$ |  |

to chemically equivalent atoms were also assumed to be chemically equivalent. Following the purpose of obtaining a better description of the chemical bonds, atomic symmetries were systematically lowered. Statistical parameters $R$ and $w R$, as well as deformation maps, served as a guide to choose the best refinement. Different atomic local symmetries were tested: $m m 2,2$ and 1 for $\mathrm{P}, \mathrm{O} 2, \mathrm{O} 4$ and $\mathrm{O} 5 ; ~ m$ and 1 for O 1 and $\mathrm{O} 3 ; m 2$ and $m$ for N atoms; cylindrical, with and without an inversion centre, for hydrogen H 4 , involved in the short hydrogen bond. The radial exponential factors $\gamma$ could not be refined along with other parameters.

In the final refinement, the chemical equivalences between different atoms were kept as those of the starting multipole refinement. The local symmetries assumed were $m m 2$ for P , $\mathrm{O} 2, \mathrm{O} 4, \mathrm{C}$ and $\mathrm{N} 1 ; m$ for O 1 and O 5 ; cylindrical plus an inversion centre for H 4 ; cylindrical for all other H atoms. The radial exponential factors were set to 2.5 for $\mathrm{P} ; 3.5$ for $\mathrm{O} 1 ; 4.5$ for $\mathrm{O} 2, \mathrm{O} 4$ and $\mathrm{O} 5 ; 5.0$ for $\mathrm{N} ; 2.5$ for $\mathrm{H} 1 ; 2.2$ for H 11 and 2.0 for H 4 . In the last cycle one scale factor, 27 positional parameters, 54 anisotropic displacement parameters and 133 coefficients of multipole functions were refined. The positional and anisotropic displacement parameters for the H atoms were fixed at their refined neutron values. The main results of this refinement, for which deformation maps are discussed, are shown in Table 1, for which deformation maps are also discussed in the next section. Atomic positions and equivalent isotropic. displacement parameters have been deposited.

## 4. Results and discussion

### 4.1. Geometrical features

Phosphorous is tetrahedally coordinated and the urea region is almost planar, as can be seen in Fig. 1. Bonds between phosphorous and hydroxyl O atoms are longer than $\mathrm{P}-\mathrm{O} 2$ and $\mathrm{P}-\mathrm{O} 4$ (Table 2). These features are the same as those observed by Konstansek \& Busing (1972).

Table 3
Hydrogen-bond parameters ( $\left(\AA,{ }^{\circ}\right.$ ).

| $A-\mathrm{H} \cdots B$ | $A \cdots B(\AA)$ | $A-\mathrm{H}(\AA)$ | $\mathrm{H}-B(\AA)$ | $A-\mathrm{H} \cdots B\left({ }^{\circ}\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{O} 1-\mathrm{H} 1 \cdots \mathrm{O} 2^{\mathrm{i}}$ | $2.649(2)$ | $1.000(1)$ | $1.643(3)$ | $174.4(3)$ |
| $\mathrm{O} 3-\mathrm{H} 3 \cdots \mathrm{O}^{\mathrm{ii}}$ | $2.589(2)$ | $1.005(1)$ | $1.584(3)$ | $177.4(3)$ |
| $\mathrm{O} 4-\mathrm{H} 4 \cdots \mathrm{O}^{2}$ | $2.410(2)$ | $1.259(1)$ | $1.160(3)$ | $170.2(3)$ |
| $\mathrm{N} 1-\mathrm{H} 11 \cdots \mathrm{O} 4^{\text {iii }}$ | $2.893(2)$ | $1.016(1)$ | $1.885(3)$ | $171.4(2)$ |
| $\mathrm{N} 1-\mathrm{H} 12 \cdots 5^{\text {iv }}$ | $3.090(2)$ | $1.004(1)$ | $2.221(3)$ | $143.9(2)$ |
| $\mathrm{N} 2-\mathrm{H} 21 \cdots \mathrm{O} 2$ | $2.943(2)$ | $1.017(1)$ | $1.936(3)$ | $170.1(2)$ |
| $\mathrm{N} 2-\mathrm{H} 22 \cdots 5^{\text {iv }}$ | $3.064(2)$ | $1.003(1)$ | $2.180(3)$ | $146.2(2)$ |

Symmetry codes: (i) $\frac{1}{2}-x,-\frac{1}{2}+y, z$; (ii) $x, \frac{1}{2}-y,-\frac{1}{2}+z$; (iii) $1-x, \frac{1}{2}+y, \frac{1}{2}-z$; (iv) $x, \frac{3}{2}-y, \frac{1}{2}+z$.

The room-temperature neutron diffraction study of $\mathrm{H}_{3} \mathrm{PO}_{4} \cdot \mathrm{CO}\left(\mathrm{NH}_{2}\right)_{2}$ shows H 4 nearer O 5 than O 4 , and Olovsson \& Jönsson (1976) classified the short hydrogen bond of the adduct as non-centred. As can be seen in Table 3, the refinement of H 4 positional parameters from the present 100 K neutron data shows a similar displacement from the centre of the bond. Thus, this indicates that H 4 is more strongly bonded to O 5 than to O 4 , in agreement with roomtemperature refinement. Fig. 1 shows the network of hydrogen bonds in the crystal.

### 4.2. Hirshfeld rigid-bond test

Correlation between electron-density distribution and atomic displacement is a problem in electron density studies. Hirshfeld (1976) postulated, as a measure of confidence in the displacement parameters, that the relative vibration of any pair of covalently bonded atoms has an effectively null component in the bonding direction. This means that $\mid(U A)^{2}-$ $(U B)^{2} \mid$ shall vanish for a pair of covalently bonded atoms $A$


Figure 2
Static deformation density maps in the planes defined by atoms (a) P, O1 and O 3 ; (b) P, O2 and O4; (c) P, O1 and O4; (d) P, O2 and O3. Contour interval $0.10 \mathrm{e}^{\circ} \AA^{-3}$; negative contours dotted and positive contours solid.

Table 4
Mean-square vibration amplitudes $\left(\AA^{2} \times 10^{4}\right)$ of bonded non-H atoms $A$ and $B$ for the rigid-bond test of vibration parameters of the final multipole refinement.

| $A$ | $B$ | $(U A)^{2}$ | $(U B)^{2}$ | $\left\|(U A)^{2}-(U B)^{2}\right\|$ |
| :--- | :--- | :--- | :--- | :--- |
| P | O1 | $97(4)$ | $100(10)$ | $3(10)$ |
| P | O2 | $75(6)$ | $78(11)$ | $3(13)$ |
| P | O3 | $77(6)$ | $77(11)$ | $0(13)$ |
| P | O4 | $84(4)$ | $86(11)$ | $2(12)$ |
| C | N1 | $94(13)$ | $95(13)$ | $1(18)$ |
| C | N2 | $92(13)$ | $85(13)$ | $7(18)$ |
| C | O5 | $97(14)$ | $89(11)$ | $8(18)$ |
| Mean |  |  |  | $3(6)$ |

and $B$, where $(U A)^{2}$ and $(U B)^{2}$ are the mean-square amplitudes of the vibration of atoms $A$ and $B$ along the $A-B$ bond.

As can be seen from Table $4,\left|(U A)^{2}-(U B)^{2}\right|$ is statistically equal to zero for all non-H $A-B$ bonds of the adduct and, consequently, the Hirshfeld postulate is satisfied. Therefore, the conclusion is that the atomic displacements are well described in the multipole refinement, and that the refinement succeeded in distinguishing between non-spherical valence electron distribution and the atomic displacements.

### 4.3. Deformation electron densities

Static deformation electron density maps are shown in Figs. 2-5. Fig. $4(b)$ and Fig. $5(f)$ also include dynamic deformation density maps, which would allow a better judgement of the data quality. Anyhow, the overall features in bonding and non-bonding regions shown in these maps are, qualitatively, similar to those found in the previous X-N study of the ureaphosphoric acid adduct at room temperature (Savage et al., 1987).
4.3.1. Phosphoric acid region. Deformation densities in $\mathrm{O}-\mathrm{P}-\mathrm{O}$ planes are shown in Fig. 2. The $\mathrm{P}-\mathrm{O} 2$ bond has a maximum peak of $0.59 \mathrm{e}^{\AA^{-3}}$. On the other hand, $\mathrm{P}-\mathrm{O} 1(\mathrm{H})$ and $\mathrm{P}-\mathrm{O} 3(\mathrm{H})$ have peak maxima of $0.44 \mathrm{e} \AA^{-3}(\mathrm{O} 1$ and O 3 were assumed to be chemically equivalent). Therefore, the peak height is higher for the $\mathrm{P}-\mathrm{O}$ bond than for $\mathrm{P}-\mathrm{O}(\mathrm{H})$ bonds, as expected from the double bond-single bond character. It is very interesting to notice the intermediate value for


Figure 3
Static deformation density maps in the planes (a) perpendicular to the plane $\mathrm{O} 2-\mathrm{P}-\mathrm{O} 4$ throughout the line of $\mathrm{P}-\mathrm{O} 2 ;(b)$ perpendicular to the $\mathrm{P}-\mathrm{O} 2$ bond, $0.1 \AA$ behind the oxygen O 2 . Contours levels defined as for Fig. 2.
the peak height of the $\mathrm{P}-\mathrm{O} 4$ bond: 0.53 e $\AA^{-3}$. This can be explained in terms of hydrogen bonding: the O 4 atom is involved in the short hydrogen bond, with $d(\mathrm{O} 4-\mathrm{H} 4)=$ 1.259 (1) $\AA$. Therefore, it is expected, for the $\mathrm{P}-\mathrm{O} 4(\mathrm{H} 4)$ bond, a bond order which is higher than a single $\mathrm{P}-\mathrm{O}(\mathrm{H})$ bond, but smaller than a double $\mathrm{P}=\mathrm{O}$ bond. These results are clearly distinct from the previous results on a crystal of phosphoric acid (Souhassou et al., 1995), where three approximately similar $\mathrm{P}-\mathrm{O}(\mathrm{H})$ interactions and a double $\mathrm{P}=\mathrm{O}$ bond are seen. In this way, it may be said that the adduct formation strengthens one $\mathrm{P}-\mathrm{O}(\mathrm{H})$ interaction and decreases the strength of the $\mathrm{P}=\mathrm{O}$ interaction of the phosphoric acid.

It is also worthwhile to comment on the electron density distribution around O 2 and O 4 . In Fig. 2(b) the plane defined by $\mathrm{P}, \mathrm{O} 2$ and O 4 shows three electron density maxima around O 4 . In this plane the $\mathrm{P}-\mathrm{O} 4$ bond, an intense lobe directed through the short hydrogen bond, and a weak lobe in the direction of a long intermolecular hydrogen bond can be seen.


Figure 4
(a) Static and (b) dynamic deformation density maps in the mean plane defined by atoms C, O5, N1 and N2. Contours levels defined as for Fig. 2.

However, as shown in Fig. 3, around O 2 there are two electron density maxima which lie in a plane orthogonal to the plane defined by $\mathrm{P}, \mathrm{O} 2$ and O 4 . Therefore, $\pi$-bond contribution between $\mathrm{P}-\mathrm{O} 2$ and $\mathrm{P}-\mathrm{O} 4$ appear to lie in perpendicular directions. These results are in agreement with those deduced theoretically by Cruickshank (1961).
4.3.2. Urea region. The deformation density map in the plane defined by the urea molecule (Fig. 4) presents two $\mathrm{C}-\mathrm{N}$ bonds with peak maxima near $0.80 \mathrm{e}^{\circ} \AA^{-3}$, the $\mathrm{C}-\mathrm{O} 5$ bond with a peak maximum of 0.73 e $\AA^{-3}$. These features are similar to those obtained in the previous deformation density study of urea crystals (Swaminathan, Craven \& McMullan et al., 1984). In urea, the two non-bonding regions are very similar, with the same peak height, probably due to the highest $\pi$-bond contribution. On the other hand, in this present study, in the $\mathrm{O} 5-\mathrm{H} 4$ direction (short hydrogen bond) the peak height is $0.71 \mathrm{e}^{\AA^{-3}}$, while the lone pair region is much more diffuse with a peak height of $0.35 \mathrm{e}^{\AA^{-3}}$ in the urea plane, probably due to the O5 environment. Figs. 5(b) and (e) show that O5 is involved in three hydrogen bonds.
4.3.3. Hydrogen bonds. Deformation densities through long hydrogen bonds are shown in Figs. 5(a)-(d). Excess charge density is found for the $\mathrm{O}-\mathrm{H}$ and $\mathrm{N}-\mathrm{H}$ covalent bonds, as well as near O atoms in the direction of the long $\mathrm{O} \cdots \mathrm{H}$ bonds. There are regions of electron depletion on the weakly bonded side of the H atoms. These features are typical for hydrogen bonds of moderate strength [see e.g. Coppens (1997), ch. 12].

The static deformation density map of the plane defined by the short hydrogen bond is shown in Fig. 5(e). The dynamic deformation density is shown in Fig. 5(f). Electron depletion near atomic positions and deformation density peaks almost midway between the two $\mathrm{O}-\mathrm{H}$ bonds, with a peak maximum of $0.71 \mathrm{e}^{\AA^{-3}}$ between O 5 and H 4 , and of $0.40 \mathrm{e}^{\AA^{-3}}$ between O 4 and H 4 . At this time, it is helpful to


Figure 5
Static deformation density maps in the planes defined by: (a) $\mathrm{N} 1-\mathrm{H} 11 \cdots \mathrm{O} 4$; (b) $\mathrm{H} 12 \cdots \mathrm{O} 5 \cdots \mathrm{H} 22 ;(c) \mathrm{N} 2-\mathrm{H} 21 \cdots \mathrm{O} 2 ;(d) \mathrm{H} 1 \cdots \mathrm{O} 2 \cdots \mathrm{H} 3 ;(e) \mathrm{O} 5 \cdots \mathrm{H} 4 \cdots \mathrm{O} 4 ;(f)$ dynamic deformation density map in the plane defined by O5 . . H4 ? O4. Contours levels defined as for Fig. 2. compare the map of the short hydrogen bond of urea-phosphoric acid (present work) with the maps of other compounds containing short symmetric hydrogen bonds, such as $\mathrm{NaH}\left(\mathrm{CH}_{3} \mathrm{COO}\right)_{2}$ (Stevens et al., 1977), $\mathrm{Na}_{5} \mathrm{H}_{3}\left(\mathrm{CO}_{3}\right)_{4}$ (Fernandes et al., 1990a), $\mathrm{KNiH}\left(\mathrm{CO}_{3}\right)_{2} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ (Fernandes et al., $\quad 1990 b), \quad\left(\mathrm{NH}_{3} \mathrm{CH}_{3}\right)\left(\mathrm{C}_{4} \mathrm{H}_{5} \mathrm{O}_{4}\right) \cdot \mathrm{H}_{2} \mathrm{O}$ (Flensburg et al., 1995) and $\left(\mathrm{NH}_{3} \mathrm{CH}_{3}\right)$ $\left(\mathrm{C}_{4} \mathrm{H}_{3} \mathrm{O}_{4}\right)$ (Madsen et al., 1998). All short hydrogen bonds present maps showing electron depletion near atomic positions and electron density peaks in bonding regions. This feature indicates some covalent character for the two $\mathrm{O}-\mathrm{H}$ interactions of each hydrogen bond, as stated by Stevens et al. (1977) for $\mathrm{NaH}\left(\mathrm{CH}_{3} \mathrm{COO}\right)_{2}$. In contrast, urea-phosphoric acid clearly shows two distinct $\mathrm{O}-\mathrm{H}$ interactions: a stronger interaction between H 4 and O 5 , with a peak height of $0.71 \mathrm{e}_{\AA^{-3}}$, and a weaker interaction between H 4 and O 4 , with a peak height
of $0.40 \mathrm{e} \AA^{-3}$. This difference is caused by the different chemical environments of O5 and O4. Refinements using the Hansen \& Coppens (1978) formalism, as implemented in the XD program (Koritsanszky et al., 1995), are also in this direction: there is one $(3,-1)$ critical point for each $\mathrm{O}-\mathrm{H}$ bond of the short hydrogen bond. The topological descriptors of the $\mathrm{O} 4-\mathrm{H} 4(3,-1)$ critical point $(\mathrm{CP})$ are $\rho=1.02 \mathrm{e}^{\AA^{-3}}, \lambda_{1}$ $=-11.18$ e $\AA^{-5}, \lambda_{2}=-11.15$ e $\AA^{5}, \lambda_{3}=11.75 \mathrm{e}^{\circ} \AA^{5}, \nabla^{2} \rho=$ $-10.58 \mathrm{e}^{-5}, d(\mathrm{O} 4-\mathrm{CP})=0.97 \AA$ and $d(\mathrm{H} 4-\mathrm{CP})=0.30 \AA$. The descriptors of the $\mathrm{O} 5-\mathrm{H} 4(3,-1) \mathrm{CP}$ are $\rho=1.20$ e $\AA^{-3}$, $\lambda_{1}=-16.87$ e $\AA^{-5}, \lambda_{2}=-16.82 \mathrm{e}^{-5}, \lambda_{3}=15.22 \mathrm{e} \AA^{-5}, \nabla^{2} \rho=$ $-18.47 \mathrm{e}^{-5}, d[\mathrm{O} 5-\mathrm{CP}]=0.94 \AA$ and $d[\mathrm{H} 4-\mathrm{CP}]=0.23 \AA$. These results reveal, in light of Bader's (1990) atoms in molecules theory, the covalent character of the two $\mathrm{O}-\mathrm{H}$ interactions of the short hydrogen bond. The Laplacians of the electron density, $\nabla^{2} \rho(\mathrm{CP})$, are negative and large in magnitude, and the electron densities, $\rho(\mathrm{CP})$, are largely positive at the $(3,-1)$ critical points of these interactions. These results are in agreement with neutron geometrical data, which show H4 nearer O5 than O4. However, considering the covalency, similar features were observed on the short symmetric hydrogen bonds of $\left(\mathrm{NH}_{3} \mathrm{CH}_{3}\right)\left(\mathrm{C}_{4} \mathrm{H}_{5} \mathrm{O}_{4}\right) \cdot \mathrm{H}_{2} \mathrm{O}$ (Flensburg et al., 1995) and $\left(\mathrm{NH}_{3} \mathrm{CH}_{3}\right)\left(\mathrm{C}_{4} \mathrm{H}_{3} \mathrm{O}_{4}\right)$ (Madsen et al., 1998). Of course, for a symmetric hydrogen bond the chemical environment is the same on both sides of the hydrogen bond.

It is helpful to compare the short $\mathrm{O} 5-\mathrm{H} 4 \cdots \mathrm{O} 4$ hydrogen bond with, for instance, the long $\mathrm{N} 2-\mathrm{H} 21 \cdots \mathrm{O} 2$ hydrogen bond, Figs. $5(c)$ and (e). The topological parameters at the $(3,-1) \mathrm{CP}$ of the $\mathrm{H} 21 \cdots \mathrm{O} 2$ interaction are $\rho=0.18 \mathrm{e}^{\AA^{-3}}, \lambda_{1}=$ $-1.16 \mathrm{e}^{-5}, \lambda_{2}=-1.14 \mathrm{e} \AA^{-5}, \lambda_{3}=3.34 \mathrm{e}^{-5}, \nabla^{2} \rho=$ $1.04 \mathrm{e}^{-5}, d(\mathrm{O} 2-\mathrm{CP})=1.27 \AA$ and $d(\mathrm{H} 21-\mathrm{CP})=0.67 \AA$. These results agree with the curves plotted by Espinosa et al. (1999) for long hydrogen bonds and enable one to characterize this hydrogen bond as an electrostatic interaction. The Laplacian of the electron density, $\nabla^{2} \rho(\mathrm{CP})$, is positive and large in magnitude and the electron density, $\rho(\mathrm{CP})$, is slightly positive at the $(3,-1)$ critical points of these interactions.

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## References

Bader, R. F. W. (1990). Atoms in Molecules. International Series of Monographs on Chemistry 22. New York: Oxford University Press.

Becker, P. J. \& Coppens, P. (1974). Acta Cryst. A30, 129-147.
Becker, P. J. \& Coppens, P. (1975). Acta Cryst. A31, 417-425.
Blessing, R. H. (1988). Acta Cryst. B44, 334-340.
Burnet, M. N. \& Johnson, C. K. (1996). ORTEPIII. Report ORNL6895. Oak Ridge National Laboratory, Tennessee, USA.

Coppens P. (1997). X-ray Charge Densities and Chemical Bonding. New York: Oxford University Press.
Cruickshank, D. W. J. (1961). J. Chem. Soc. pp. 5486-5504.
Dovesi, R., Causa, M., Orlando, R., Roetti, C. \& Saunders, V. R. (1990). J. Chem. Phys. 92, 7402-7411.

Espinosa, E., Souhassou, M., Lachekar, H. \& Lecomte, C. (1999). Acta Cryst. B55, 563-572.
Fernandes, N. G., Tellgren, R. \& Olovsson, I. (1990a). Acta Cryst. B46, 458-466.
Fernandes, N. G., Tellgren, R. \& Olovsson, I. (1990b). Acta Cryst. B46, 466-474.
Flensburg, C., Larsen, S. \& Stewart, R. F. (1995). J. Phys. Chem. 99, 10686-10696.
Hansen, N. K. \& Coppens, P. (1978). Acta Cryst. A34, 909-921.
Harel, M. \& Hirshfeld, F. L. (1975). Acta Cryst. B31, 162-171.
Hirshfeld, F. L. (1971). Acta Cryst. B27, 769-781.
Hirshfeld, F. L. (1976). Acta Cryst. A32, 239-244.
Hirshfeld, F. L. (1977). Isr. J. Chem. 16, 226-229.
Konstansek, E. C. \& Busing, W. R. (1972). Acta Cryst. B28, 24542459.

Koritsanszky, T., Howard, S., Richter, T., Su, Z., Mallinson, P. R. \& Hansen, N. K. (1995). XD. Freie University, Berlin.
Lundgren, J. O. (1982). Crystallographic Computer Programs, Report UUIC-B13-04-05. Institute of Chemistry, University of Uppsala, Sweden.
Madsen, D., Flensburg, C. \& Larsen, S. (1998). J. Phys. Chem. A, 102, 2177-2188.
Maslen, E. N., Fox, A. G. \& O'Keefe, M. A. (1995). International Tables for Crystallography, edited by A. J. C. Wilson, Vol. C, pp. 476-512. Dordrecht: Kluwer Academic Publishers.
Moss, G. R., Souhassou, M., Blessing, R. H., Espinosa, E. \& Lecomte, C. (1995). Acta Cryst. B51, 650-660.

Motz, D. \& Albrand, K.-R. (1972). Acta Cryst. B28, 2459-2463.
Olovsson, I. \& Jönsson, P.-G. (1976). The Hydrogen Bond - Recent Developments in Theory and Experiment, edited by P. Schuster, G. Zundel and C. Sandorfy, Vol. II, ch. 8. Amsterdam: North-Holland.
Savage, F. J., Blessing, R. H. \& Wunderlich, H. (1987). Trans. Am. Cryst. Assoc. 23, 97-100.
Sears, V. F. (1995). International Tables for Crystallography, edited by A. J. C. Wilson, Vol. C, pp. 383-391. Dordrecht: Kluwer Academic Publishers.
Siemens (1991). XSCANS User's Manual. Siemens Analytical X-ray Instruments Inc., Madison, Wisconsin, USA.
Souhassou, M., Espinosa, E., Lecomte, C. \& Blessing, R. H. (1995). Acta Cryst. B51, 661-668.
Stevens, E. D., Lehmann, M. S. \& Coppens, P. (1977). J. Am. Chem. Soc. 99, 2829-2831.
Swaminathan, S., Craven, B. M. \& McMullan, R. K. (1984). Acta Cryst. B40, 300-306.
Swaminathan, S., Craven, B. M., Spackman, M. A \& Stewart, R. F. (1984). Acta Cryst. B40, 398-404.

Zavodnik, V., Stash, A., Tsirelson, V., De Vries, R. \& Feil, D. (1999). Acta Cryst. B55, 45-54.


[^0]:    ${ }^{\mathbf{1}}$ Supplementary data for this paper are available from the IUCr electronic archives (Reference: CA0006). Services for accessing these data are described at the back of the journal.

